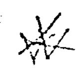
Al-Fateh University, Faculty of Engineering
Electrical and Electronics Engineering Department
EE 303 Numerical Techniques and Programming
Final Examination, July 9th, 2009

(1) Answer all questions to the best of your knowledge (2) No question will be answered during the exam (3) Show all your steps and carry all computations to 4 decimal places unless otherwise mentioned.

(12.5 Marks for each question)



Q1- Using Simpson's $\frac{1}{3}$ rule for double integration, evaluate the following integral and compare your answer to the analytical solution in term of relative error. (*Use* $\Delta x=0.3$, $\Delta y=0.2$)

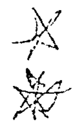
$$\int_{-0.2}^{1.0}\int_{0.4}^{1.2} e^{x}\cos(2y)\,dy\,dx$$

Q2- Solve $\frac{dy}{dx} = \sin(x) + y$, $y(0) = 2$, analytically then using simplified and improved Euler's methods. Check your answers by finding the relative error to the analytical solution. *Use h=0.25 and tabulate your answers as follows :*

x	$y(x)$, Simplified Euler	$y(x)$, Improved Euler	$y(x)$, Analytical Answer	Relative Error	
				Simplified	Improved
0.0					
0.25					
0.5					
0.75					
1.00					

Q3- Solve $y' = 2x^2 - y$, $y(0) = -1$, analytically then using second order Runge-Kutta *(Heuns' method)*, compare your answer with the fourth order Kutta's method. *Use h=1 and tabulate your answers as follows:*

x	$y(x)$ 2nd order	$y(x)$ 4th Order	Analytical Answer	2nd order Relative Error	4th order Relative Error
0					
1					
2					



Q4- Given the following matrix

$$\begin{bmatrix} 2.1 & x \\ 3x & 3.5 \end{bmatrix}$$

Find the value of x that will make the condition number of this matrix approximately 100.0. Carry all calculations to 5 decimal places.

Good luck to all of you.

$-\frac{\cos x}{2} - \frac{\sin x}{2} + e^{x}$

Al-Fateh University
Faculty of Engineering
Electrical and Electronics Engineering Department

EE 303 Numerical Techniques and Programming

Midterm II, January 7th, 2009

- Answer **any two questions** to the best of your knowledge.
- Carry all calculations to 4 digits
- No questions will be answered during the exam.

Time allowed: 2. *hours*

Q1- Given a matrix of the form Ax=b where

$$A = \begin{bmatrix} 2.9800 & 1.7600 & -5.0500 \\ 3.7800 & 8.1200 & 0.7600 \\ 2.0223 & 4.3442 & 0.4071 \end{bmatrix}, \quad b = \begin{bmatrix} -300.7336 \\ 2254.2222 \\ 1206.0751 \end{bmatrix}$$

(a) If vector b is changed by (-10%), what is the relative error introduced to the solution vector of this system? (5 Marks)

(b) Given $A = \begin{bmatrix} a & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & a \end{bmatrix}$

(1) What is the value of a that will make this matrix singular? (1 Mark)

(2) What is the value of a that will make the condition number of this matrix > 100? (2 Marks)

(3) Find the Eigen values of the system in term of a? (2 Marks)

Q2- For the Following data pairs:

x	0.3	0.5	0.7	0.9	1.1
y	0.40496	0.82436	1.40963	2.21363	3.30458

(a) Construct a Neville table that interpolates at x=0.6 using only the first four points. (3 Marks)

(b) Fit a 3rd degree polynomial of the form $y = a_0 + a_1x + a_2x^2 + a_3x^3$ using Non- linear Least Square and use it to estimate $f(0.6)$ (3 Marks)

(c) Estimate $f(0.6)$ using 3rd degree Newton-Gregory interpolating polynomial (3 Marks)

(d) The table is for $f(x) = x\,e^{x}$. Which method gave the closest answer to the correct answer. (1 Mark)

Q3- The following table is for e^{x^2}

x	0.0	0.4	0.9	1.5	1.8
$f(x)$	1.0000	1.1735	2.2479	9.4877	25.5337

(a) Find the first 4 parameters of the fitting polynomial of the form: (3 Marks)

$$P_n(x) = a_0 + \sum_{i=1}^{n} a_i \prod_{i=0}^{n}(x - x_i)$$

(b) Write down the divide difference tables to: (2 Marks)

(1) Estimate $e^{1.2}$ using the first four points (2 Marks)

(2) Estimate $e^{1.2}$ using the last four points

(c) Write a 4th degree Lagrange interpolating polynomial that interpolates the given data and use it to estimate $f(1.2)$ (2 Marks)

(d) Which method gave the best estimate? (1 Mark)

Good luck to all of you

Alfateh University

Electrical Engineering Department

EE303 Numerical Analysis

Mid-Term II

Answer all questions, *Carry calculations to 3 decimal places,* *time allowed 2 hours*

(Q1) Let $A = \begin{bmatrix} [illegible] & [illegible] & [illegible] \\ 1 & -8 & 12 \\ 2 & 11 & -1 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $b = \begin{bmatrix} 73 \\ 137 \\ -65 \end{bmatrix}$

a) Solve the system using Gaussian elimination without pivoting (5 Marks)

b) By using the Gaussian elimination coefficients find the LU matrices. (5 Marks)

(Q2) Give the following system of linear equations

$12x_1 - 4x_2 + 5x_3 = 59$

$4x_1 - 18x_2 + 2x_3 = 172$

$5x_1 + 14x_2 - 10x_3 = -201$

a) Solve the system using *LU decomposition* with the following two stages (8 Marks)

$Lz = b$ solve for z

$Ux = z$ solve for x

b) Show that (determinant) $det(A) = det(L) * det(U)$ (2 Marks)

a) Use nonlinear least square fitting to find a polynomial of degree 2 to fit the following tabulated data then find $f(1.0)$ (5 Marks)

x	-3	-1.9	-0.8	0.3	1.4	2.5	3.6	4.7	5.8
$f(x)$	19	-0.525	-14	-21.425	-22.8	-18.125	-7.4	9.375	32.2

b) Fit a fourth order polynomial through the first five points using Lagrangian polynomial and find $f(1.0)$ (5 Marks)

Good Luck to all of you

Al-Fateh University

EE303 Numerical Techniques & Programming

Final Examination

Time Allowed: 3 Hours (Answer all questions) each question is 10 Marks

Q1. Using Newton's method, find the root of the following non-linear function $f(x) = e^{-x^2} - \sin(2x)$ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ repeat until the error $\leq 10^{-5}$

Q 2. Using non-linear least square fitting, find a polynomial of degree 3 to fit the following data

X	0.5000	0.8000	1.0000	1.3000	1.6500	2.0000	2.3000	3.0000
F(x)	-2.6500	0.3920	2.9000	7.7420	15.6091	26.6000	39.0920	82.1000

b) Find the value of the function at x=1.5

Q3. a) Using the trapezoidal method, find the numerical integration of the function represented by the following table.

X	-1.0	-0.7	-0.4	-0.1	0.2	0.5	0.8	1.1	1.4	[illegible]	[illegible]
F(x)	2.7183	1.4092	1.0661	1.0010	0.9920	0.8825	0.5993	0.2642	0.0643	0.0074	0.00[illegible]3

b) Repeat using Simpson's rule

Q.4 Use composite Simpson's rule to approximate the following multiple integrals

$$\int_{1.4}^{2.0}\int_{1.0}^{1.5} \sin(x+2y)\,dy\,dx$$

with n=m=4

Q5. The following ordinary differential equation

$y' = y - t^2 + 1, \quad 0 \leq t \leq 2, \quad y(0) = 0.5$ has an exact solution $y(t) = (t+1)^2 - \frac{e^t}{2}$

a) Use Euler's method with N=10 to approximate the solution

b) Use Taylor series of order 4 and compare the results with the results obtained in part a) of this question and the analytical values

Good Luck to All of You

Al-Fateh University

Faculty of Engineering- Electrical & Electronics Engineering Department

EE303 Numerical Techniques & Programming (Final Examination)

Time Allowed: 3 Hours (Answer all questions) each question is 10 Marks

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Q1. a) Using the Trapezoidal rule, find the numerical integration of the function represented by the following table.

x	-1.0	-0.7	-0.4	-0.1	0.2	0.5	0.8	1.1	1.4	1.7	2.
$f(x)$	2.7183	1.4092	1.0661	1.0010	0.9920	0.8825	0.5993	0.2642	0.0643	0.0074	0.0003

b) Repeat using Simpson's rule

Q2 - Given the following data

x	0.5	-0.2	0.7	0.1	0.0
$f(x)$	1.0025	1.394	1.0084	1.0221	1.1884

Use the divide difference to estimate $f(0.15)$ using a polynomial of degree 3 through the first 4 points

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Q3- Use Newton-Gregory forward interpolating polynomial of degree 3 to estimate $f(0.75)$

x	0.00	0.2	0.4	0.6	0.8	1.0	1.2	1.4
$f(x)$	3.000	2.4560	1.6880	0.7920	-0.1360	-1.0	-1.7040	-2.1520

(Hint) $P_n(x_s) = f_0 + \binom{S}{1}\Delta f_0 + \binom{S}{2}\Delta^2 f_0 + \binom{S}{3}\Delta^3 f_0 \dots\dots\dots$

Q4 – Using the Simpson's $\frac{1}{3}$ rule for multiple integration, find the value of following

$\int_{-0.2}^{1.4}\int_{0.4}^{2.6} e^{y} \cos(x)\, dy\, dx$

Q5- Estimate the error between $x=1$ and $x=1.6$ when the simple Euler method is used to solve

$\frac{dy}{dx} = xy^2$, $y(1)=1$. *The analytical solution is* $y=\frac{3}{3-x^3}$

a) Use h=0.1 Compute to the actual errors at each step with the analytical solution.

b) Use Taylor series of order 4 and compare the results with the results obtained in part (a) of this question and the analytical values.

Good luck to all of you

Faculty of Engineering
Electrical and Electronics Engineering Department
EE 303 Numerical Techniques and Programming
Midterm II, December 14th, 2009

a) Answer all questions to the best of your knowledge.
b) Show all steps and carry all calculations up to 4 digits unless otherwise mentioned.
c) No question will be answered during the exam.
d) Time allowed: 2 hours

Q1-
(a) Derive Simpson's 1/3 rule by integrating the second-degree Newton-Gregory polynomial that fits *f(x)* at *x*-values of x_1, x_2, x_3 which are evenly spaced at distance *h* apart. (5 Marks)

(b) Write a C program for computing numerical integration using the trapezoidal method. (3 Marks)

(c) Use *3h/8* Simpson's rule to evaluate the following integral:

$$\int_{-0.4}^{1.4} \sin(2x)\,dx$$

Use Δx=0.3 (5 Marks)

(d) What is the relative error of the solution obtained in part (C) ? (2 Marks)

Q2. Using the following data

x	-0.243	1.123	0291	0.423	0.789	1.245	2.346
f(x)	0.8153	0.8918	1.5877	2.1087	4.8583	12.1478	58.2974

(a) Using nonlinear least square fitting, find a 3rd degree polynomial and use it to estimate *f(0.5)*. (6 Marks)

(b) Using forward divide difference, find a 5th degree interpolating polynomial and use it to estimate *f(0.5)*. (6 Marks)

(c) Given the true value *f(0.5)*=2.5125, calculate the relative error in both cases and identify which method is more accurate. (3 Marks)

Good luck to all of you

Al-Fateh University

Faculty of Engineering

Electrical and Electronics Engineering Department

EE 303 Numerical Techniques and Programming

Quiz, December 24th,2008

- Answer any two questions to the best of your knowledge. Time allowed: **2. hours**

Q1- Given a matrix of the form Ax=b where

$$A = \begin{bmatrix} 3.2000 & -1.0500 & 2.0530 \\ 4.2656 & -1.3997 & 2.7000 \\ -0.0830 & -0.0340 & 1.0470 \end{bmatrix}, \quad b = \begin{bmatrix} -2.2440 \\ -3.0645 \\ 2.0750 \end{bmatrix}$$

(a) Write a c/c++ code for dynamic memory allocation for any matrix A of type float with columns =m and rows=n. (1 Mark)

(b) Write two Matlab. commands for finding the condition number of matrix A. (1 Mark)

(c) Write the formula for the Eigen values of Matrix A(*do not find the values*). (2 Marks)

(d) Using two methods outlined in class, show that the matrix is ill-conditioned. (6 Marks)

Q2- For the Following data pairs:

x	1.2	1.8	2.5	3.6
y	2.847	1.680	0.039	0.0045

(a) Write a Matlab. Command for fitting a polynomial of 3rd degree and for evaluating the same polynomial at x= 2.0. (2 Marks)

(b) Write a Lagrange interpolation polynomial that interpolates the given data and used it to estimate $f(1.5)$ and $f(2.0)$. (4 Marks)

(c) Compare your results with 3rd degree Non-Linear Least Square method. (4 Marks)

Q3- Write down the divide difference table for e^x using the values

x	e^x
0.0	1.0000
0.4	1.49182
0.9	2.4596
1.5	4.4817
1.8	6.0496

(a) Find the first 3 parameters of the fitting polynomial of the form: (3 Marks)

$P_n(x) = a_0 + \sum_{i=1}^{n} a_i \prod_{i=0}^{n}(x - x_i)$

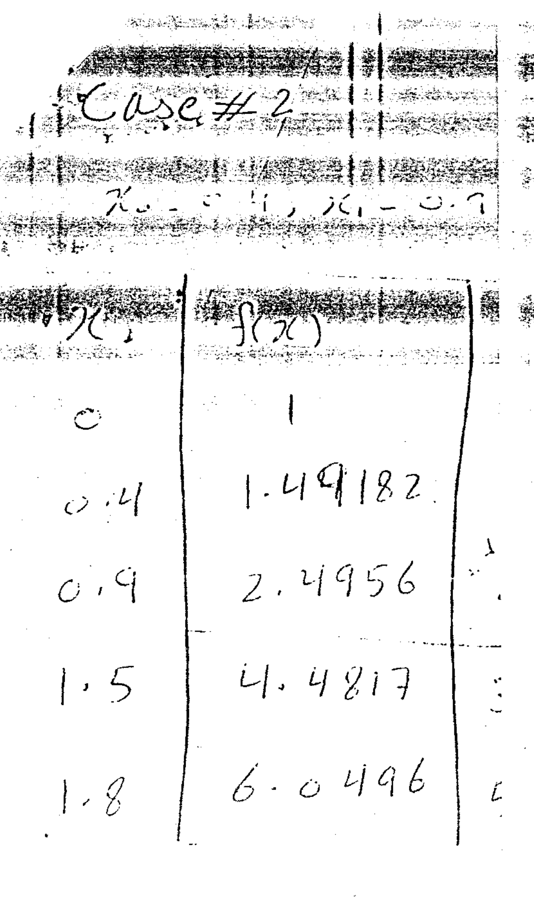
(b) Estimate $e^{1.2}$ using (7 Marks)

1-Cubic interpolator with $x_0 = 0.0$

2-Cubic interpolator with $x_0 = 0.4$

Which gives a better estimate?

Good luck to all of you



Case # 2

$x_0 = 0.4,\ x_1 = 0.9$

x_i	$f(x)$
0	1
0.4	1.49182
0.9	2.4956
1.5	4.4817
1.8	6.0496

$\therefore f(x) = 1.49$

$f(x) = 3.33$

$\varepsilon = 0$

Ex:

x	$f(x) = e^x$
0	1
0.4	1.49182
0.9	2.4956
1.5	4.4817
1.8	6.0496

Using N.D.D polynomial Inter

Estimate $e^{1.2}$ using

① Cubic Interpolation with $x_0 =$
$x_0 =$

② " " "

Solution

$\because f(x) = a_0 + a_1(x - x_0) + a_2(x - x$

Case # 1

x	$f(x)$	1^{st} D.D	2^{nd} D.D
0	1		
0.4	1.49182	1.22955	
0.9	2.4956	2.00756	0.86
1.5	4.4817	1.9861	-0.01
1.8			

$f(x) = 1 + 1.22955(x) + 0.8644$
$- 0.5893$

$\therefore I = 3.36899$

$f(x) = 1 + 1.22955(x) + 0.86446(x \cdot (x$

$f(x) = 3.5543$

$\varepsilon = 7.05\ \%$